Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/26**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. Three particles of masses $4m$, $2m$ and km are placed at the points with coordinates $(-3,-1)$, $(6, 1)$ and $(-1, 5)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(\bar{x}, \bar{y})$

(a) show that $\bar{x} = \dfrac{-k}{k+6}$

(3)

(b) find $\bar{y}$ in terms of k.

(2)

Given that the centre of mass of the three particles lies on the line with equation $y = 2x + 3$

(c) find the value of k.

(2)

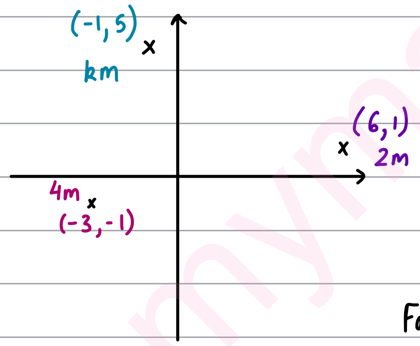
A fourth particle is placed at the point with coordinates $(\lambda, 4)$.

Given that the centre of mass of the four particles also lies on the line with equation $y = 2x + 3$

(d) find the value of λ.

(2)

1) a) Firstly consider moments about the y-axis, imagining the masses on a 2-D plane. → perpendicular distance from y axis = x-coordinate



moments = force x perpendicular distance

The sum of moments is equal to the overall moment acting through the COM.

Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$

where m = force

x = perpendicular distance

For $\bar{x}$, we find the COM from the y axis. Applying formulae:

$$4mg \times (-3) + 2mg \times (6) + kmg(-1) = (4m + 2m + km)g\,\bar{x}$$

Cancelling out g and m from each term and simplifying, we obtain

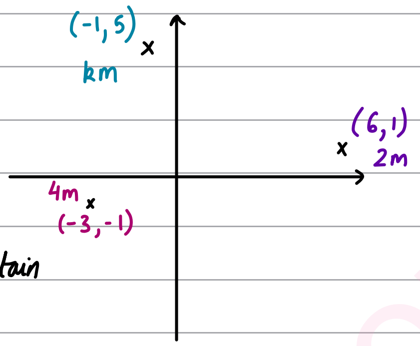
$$-12 + 12 - k = (6+k)\bar{x}$$

Rearranging, we get

$$\boxed{\bar{x} = \frac{-k}{6+k}}$$

Question 1 continued

b) Repeating the same method as part a, for $\bar{y}$, we find the COM from the x axis by considering moments about the x axis.



$$4mg \times (-1) + 2mg \times (1) + kmg \times 5 = (4m + 2m + km)g\,\bar{y}$$

Cancelling out g and m from each term and simplifying, we obtain

$$-4 + 2 + 5k = (6+k)\bar{y}$$

Rearranging, we get

$$\bar{y} = \frac{5k-2}{6+k}$$

c) The COM is $(\bar{x}, \bar{y})$ which we know as $\left(\frac{-k}{6+k}, \frac{5k-2}{6+k}\right)$ in terms of k

Since the COM lies on $y = 2x+3$, substitute the $\bar{x}$ and $\bar{y}$ values into the equation

$$y = 2x + 3$$

$$\frac{5k-2}{6+k} = 2\left(\frac{-k}{6+k}\right) + 3$$

Multiply both sides by $(6+k)$

$$5k - 2 = -2k + 3(6+k)$$

$$5k - 2 = -2k + 18 + 3k$$

$$4k = 20$$

$$k = 5$$

d) Adding the 4th particle does not change the COM from the line $y = 2x+3$ which means that the 4th particle must also lie on that line for the COM to not change from that line.

Substituting the coordinates of the 4th particle $(\lambda, 4)$ into $y = 2x+3$

$$4 = 2\lambda + 3$$

$$1 = 2\lambda$$

$$\lambda = \frac{1}{2}$$

Question 1 continued

Question 1 continued

(Total for Question 1 is 9 marks)

2. A particle P is moving along the x-axis.
At time t seconds, $t \geqslant 0$, P has acceleration $a\,\mathrm{m\,s^{-2}}$ and velocity $v\,\mathrm{m\,s^{-1}}$ in the direction of x increasing, where

$$v = e^{2t} + 6e^{t} - kt$$

and k is a positive constant.

When $t = \ln 2$, $a = 0$

(a) Find the value of k.

(4)

When $t = 0$, the particle passes through the fixed point A.
When $t = \ln 2$, the particle is d metres from A.

(b) Showing all stages of your working, find the value of d correct to 2 significant figures.
[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

a) acceleration = change in velocity per unit time

$a = \frac{dv}{dt}$ ↳ the same as the derivative of velocity mathematically

To find a in terms of t, apply formulae.

$v = e^{2t} + 6e^{t} - kt$

$a = \frac{dv}{dt} = 2e^{2t} + 6e^{t} - k$

Rules: $\frac{d}{dt}(e^{t}) = e^{t}$

$\frac{d}{dt}(e^{kt}) = ke^{kt}$

Substituting values $t = \ln 2$ and $a = 0$

Log Law: $e^{a\ln b} = e^{\ln ab}$

$e^{\ln a} = a$

$2e^{2(\ln 2)} + 6e^{\ln 2} - k = 0$

$2e^{\ln 4} + 6e^{\ln 2} - k = 0$

$2(4) + 6(2) - k = 0$

$k = 20$

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Question 2 continued

b) At $t=0$, displacement from $A=0$ $(x=0)$

At $t=\ln 2$, displacement from $A=d$ $(x=d)$

Let x be formulae for displacement from A

Velocity = change in displacement per unit time

$v = \frac{dx}{dt} \Longrightarrow x = \int v \, dt$

integrate both sides

To find x in terms of t, apply formulae.

using k value from part b

$x = \int v \, dt = \int (e^{2t} + 6e^{t} - 20t) \, dt$

$= \frac{1}{2}e^{2t} + 6e^{t} - 10t^2 + c$

To find 'd', use boundary conditions of $t=0$ and $t=\ln 2$

$d = \int_0^{\ln 2} e^{2t} + 6e^{t} - 20t \; dt$

$= \left[\frac{1}{2}e^{2t} + 6e^{t} - 10t^2\right]_0^{\ln 2}$

$= \frac{1}{2}e^{2\ln 2} + 6e^{\ln 2} - 10(\ln 2)^2 - \left(\frac{1}{2}e^{0} + 6e^{0} - 10(0)^2\right)$

$= \frac{1}{2}(4) + 6(2) - 10(\ln 2)^2 - \frac{13}{2}$

$= \frac{15}{2} - 10(\ln 2)^2$

$d \approx 2.7 \quad (2sf)$

(Total for Question 2 is 8 marks)

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Turn over

3. A girl is cycling round a circular track.
The girl and her bicycle have a combined mass of 55 kg.
The coefficient of friction between the track surface and the tyres of the bicycle is μ.

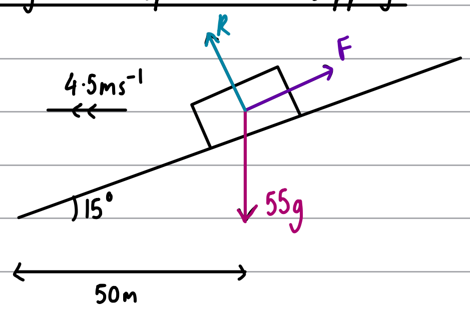
The track is banked at an angle of 15° to the horizontal.

The girl and her bicycle are modelled as a particle moving in a horizontal circle of radius 50 m
The minimum speed at which the girl can cycle round this circle without slipping is 4.5 m s^{-1}

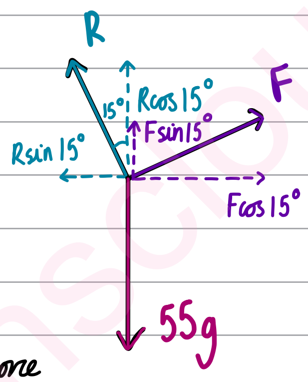
Using the model, find the value of μ.

(9)

Car travelling at max speed without slipping



Resolving forces horizontally and vertically:



By considering horizontal motion and resultant force

centripetal force = horizontal normal contact force − horizontal friction

resultant force

$$\frac{mv^2}{r} = R\sin 15° - F\cos 15°$$

Formulae for friction is $F = \mu R$ where μ = friction coefficient, R = normal contact force

$$\frac{55 \times 4.5^2}{50} = R\sin 15° - F\cos 15°$$

Substituting $F = \mu R$

$$22.275 = R(\sin 15° - \mu\cos 15°) \quad ②$$

Resolving forces vertically

$$R\cos 15 + F\sin 15 = 55g$$

Substituting $F = \mu R$ and using $g = 9.8$

$$R(\cos 15° + \mu \sin 15°) = 539$$

$$R = \frac{539}{\cos 15° + \mu\sin 15°} \quad ①$$

Substituting ① into ②

$$22.275 = \frac{539(\sin 15° - \mu\cos 15°)}{\cos 15° + \mu \sin 15°}$$

Rearranging

$$\frac{81}{1960}\cos 15° + \frac{81}{1960}\mu\sin 15° = \sin 15° - \mu\cos 15°$$

$$\mu\left(\frac{81}{1960}\sin 15° + \cos 15°\right) = \sin 15° - \frac{81}{1960}\cos 15°$$

$$\mu = \frac{\sin 15° - \frac{81}{1960}\cos 15°}{\frac{81}{1960}\sin 15° + \cos 15°} \approx 0.224 \text{ (3sf)}$$

$$\therefore \mu = 0.224$$

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

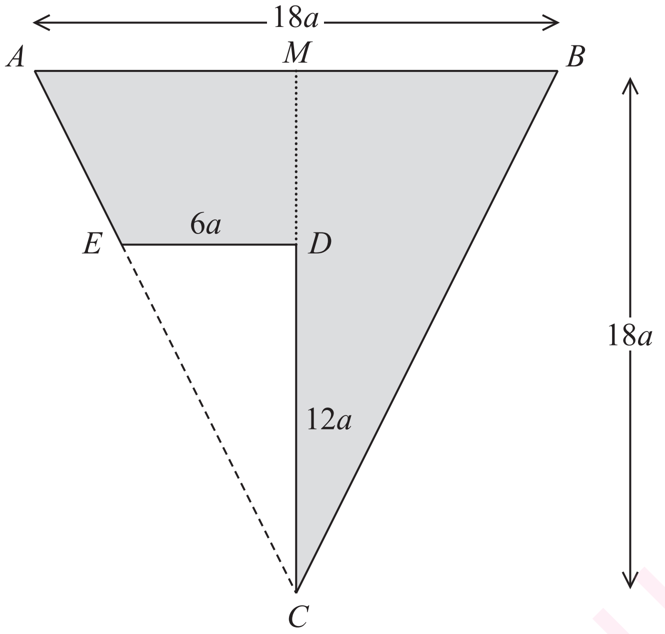


Figure 1

A uniform triangular lamina ABC is isosceles, with $AC = BC$. The midpoint of AB is M. The length of AB is $18a$ and the length of CM is $18a$.

The triangular lamina CDE, with $DE = 6a$ and $CD = 12a$, has ED parallel to AB and MDC is a straight line.

Triangle CDE is removed from triangle ABC to form the lamina L, shown shaded in Figure 1.

The distance of the centre of mass of L from MC is d.

(a) Show that $d = \frac{4}{7}a$

(4)

The lamina L is suspended by two light inextensible strings. One string is attached to L at A and the other string is attached to L at B.
The lamina hangs in equilibrium in a vertical plane with the strings vertical and AB horizontal.
The weight of L is W

(b) Find, in terms of W, the tension in the string attached to L at B

(3)

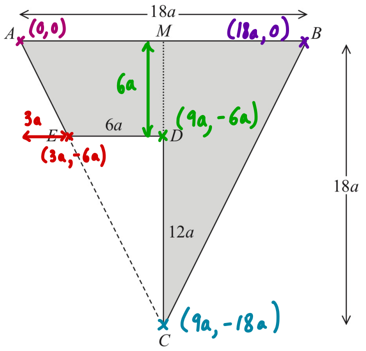
The string attached to L at B breaks, so that L is now suspended from A.
When L is hanging in equilibrium in a vertical plane, the angle between AB and the downward vertical through A is $\theta°$

(c) Find the value of θ

(7)

Question 4 continued

a)



Taking Point A with coordinates (0,0) as the origin

COM of big Isosceles triangle distance from A in x direction

$\hookrightarrow \dfrac{0 + 18a + 9a}{3} = 9a$ (Also can be figured out as along line of symmetry)

COM of small triangle cut out distance from A in x direction

$\hookrightarrow \dfrac{3a + 9a + 9a}{3} = 7a$

Overall COM of lamina L

(Area of big triangle x COM) − (Area of cut-out triangle x COM) = Area of lamina L x COM

Since Lamina L is a shape with a cut out, we can use the moment of the larger shape without the cutout minus the moment of the cutout to obtain the moment of the Lamina L. The moment of the lamina is the area of the lamina x COM so it can be determined by rearranging.

$$\frac{1}{2}(18a)(18a) \times 9a - \frac{1}{2}(6a)(12a) \times 7a = \left(\frac{1}{2}(18a)(18a) - \frac{1}{2}(6a)(12a)\right) \times \bar{x}$$

$$1458a^3 - 252a^3 = 126a^2\bar{x}$$

$$1206a^3 = 126a^2\bar{x}$$

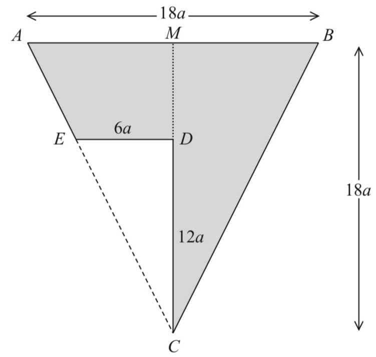
$$\bar{x} = \frac{1206a^3}{126a^2} = \frac{67}{7}a$$

To find distance of COM from MC, do $\dfrac{67}{7}a - \underbrace{9a}_{\text{distance from A to MC}} = \dfrac{4}{7}a$

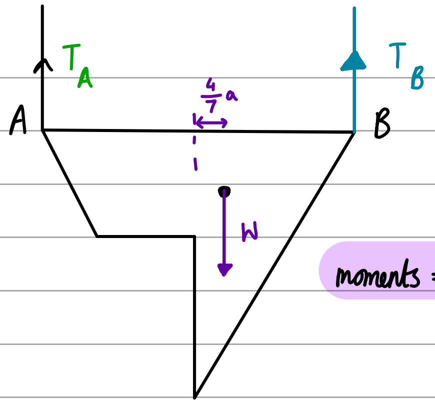
$\therefore$ distance from $MC = \dfrac{4}{7}a$

Question 4 continued

b)



$\Rightarrow$



moments = force x perpendicular distance

Taking moments about the point A

Sum of clockwise moments = sum of anti-clockwise moments

$\frac{67}{7}a \times W = 18a \times T_B$

[Re-arranging] $T_B = \frac{67}{126} W$

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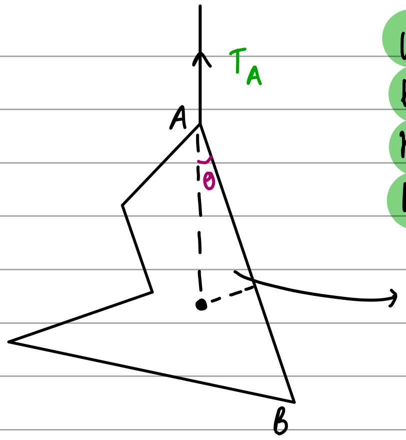
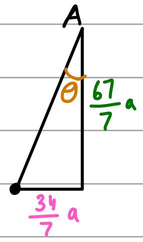
Question 4 continued

c)



When string at B breaks, the downward vertical following the line of string at A goes through the COM



Firstly, the height of triangle is the distance of the COM from A which from part a) is $\frac{67}{7}a$

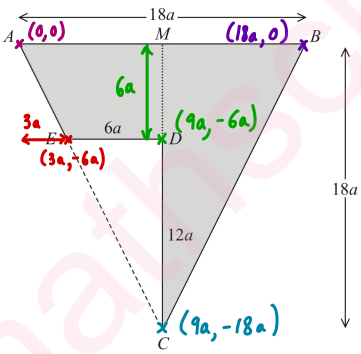
Using trigonometry

$\tan\theta = \dfrac{\frac{34}{7}a}{\frac{67}{7}a} = \dfrac{34}{67}$

$\theta = \tan^{-1}\left(\frac{34}{67}\right) = 26.9^\circ$ (3sf)

$\therefore$ $\theta = 26.9^\circ$ (3sf)

To find the width, we must find the distance of the COM from AB so we must use the same process as part a) in the y direction.



COM of big Isosceles triangle distance from A in y direction

$\hookrightarrow \dfrac{0 + 0 + -18a}{3} = -6a$

COM of small triangle cut out distance from A in y direction

$\hookrightarrow \dfrac{-6a + -6a + -18a}{3} = -10a$

Overall COM of lamina L

(Area of big triangle × COM) − (Area of cut-out triangle × COM) = Area of lamina L × COM

$\frac{1}{2}(18a)(18a) \times -6a - \frac{1}{2}(6a)(12a) \times -10a = \left(\frac{1}{2}(18a)(18a) - \frac{1}{2}(6a)(12a)\right) \times \bar{y}$

$-972a^3 + 360a^3 = 126a^2\bar{y}$

$-612a^3 = 126a^2\bar{y}$

$\bar{y} = -\frac{34}{7}a$

$\therefore$ COM from AB $= \frac{34}{7}a$

Question 4 continued

(Total for Question 4 is 14 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS